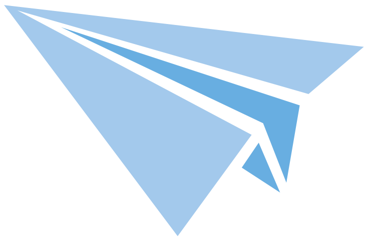
babcock™

Tessellation and Miura Folds

15-30 MINUTES

Materials

- 1+ sheets of A4 paper (210 x 297 mm)
- A ruler
- Coloured pens (optional)
- A protractor



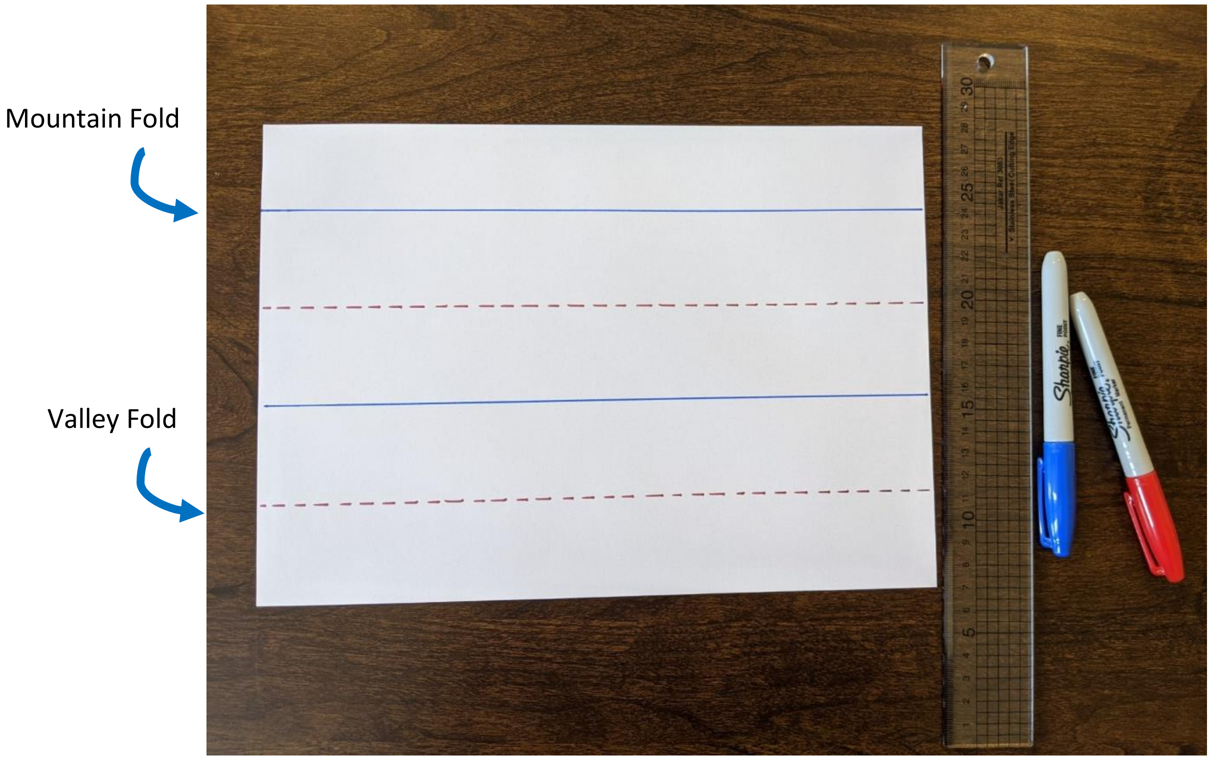
Activity Overview

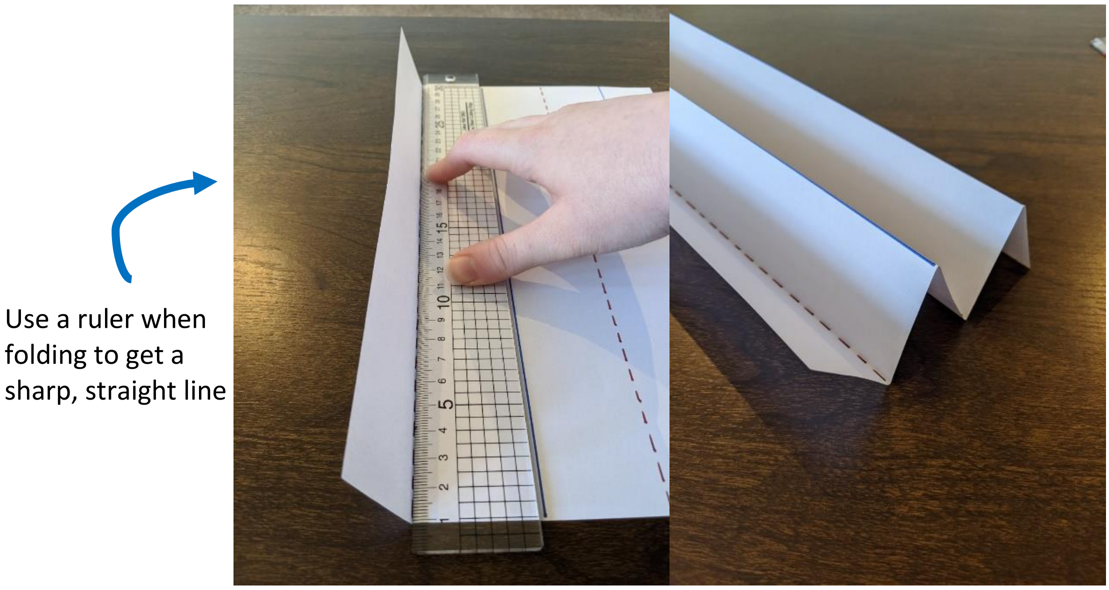
› Learn how to create your own Miura folds (method of folding a flat surface, such as a sheet of paper, into a smaller area). The fold is named after its inventor, Japanese astrophysicist Koryo Miura. Use a simple piece of paper before reverse engineering your own design for a different sized piece of paper with your own observations.

› Activity from: https://www.sciencefriday.com/educational-resources/tessellation-and-miura-folds/

Activity Plan

› Fold the paper into five evenly spaced sections, alternating mountain and valley folds to create an accordion fold – also called a concertina fold.

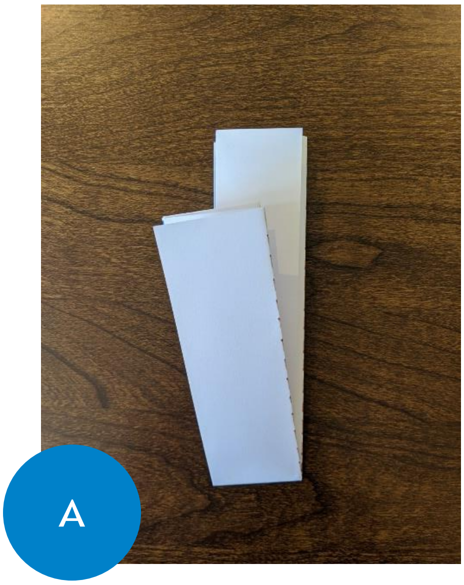


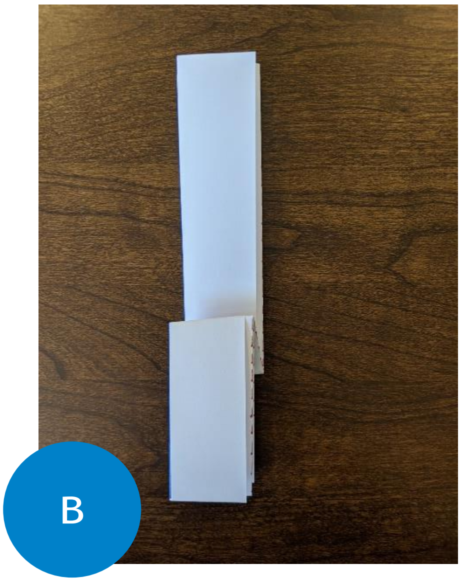


Pattern of mountain and valley folds to create your five columns

› Keeping the paper folded (it should look like a tall, thin rectangle), you will create seven more sections by alternating mountain and valley folds. To make the first fold, take the bottom of the paper and fold a section up and at an angle to the left, so that the top left corner is about 4cm from the top of the rest of the paper and 2cm from the left edge of the rest of the paper (see picture A).

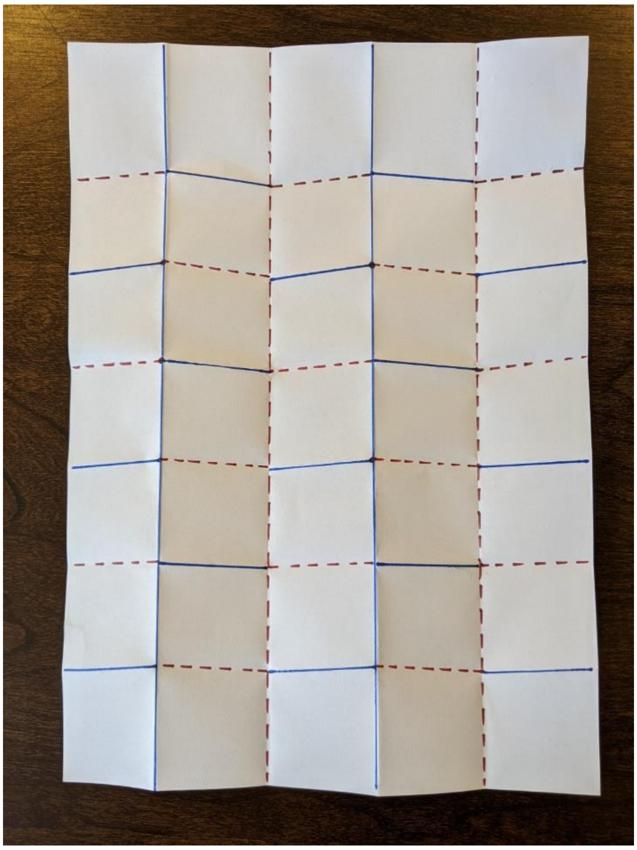
› Next, fold a portion of the paper that you just folded up, back down so that its front right edge is parallel with the right edge of what remains of the original tall, thin rectangle (pictures B+C). You have created your first section. Continue this process until you've created seven sections (you will have to turn the paper over at some point. See the diagram below and the video further down for visual guides.)





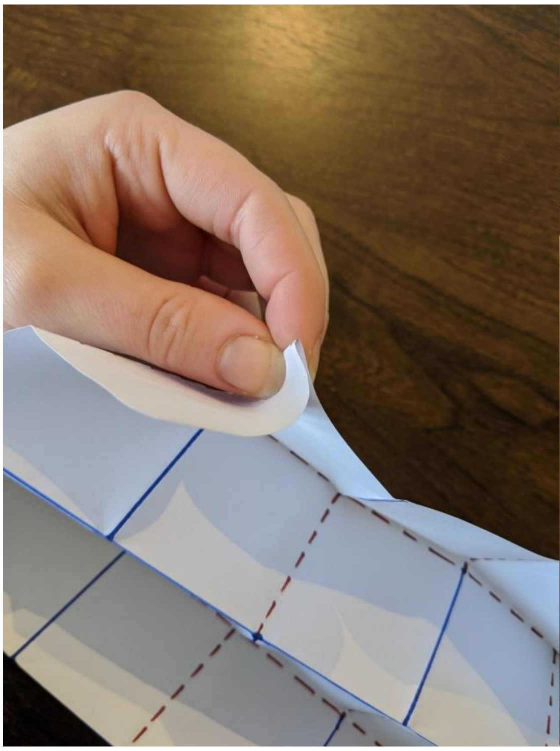
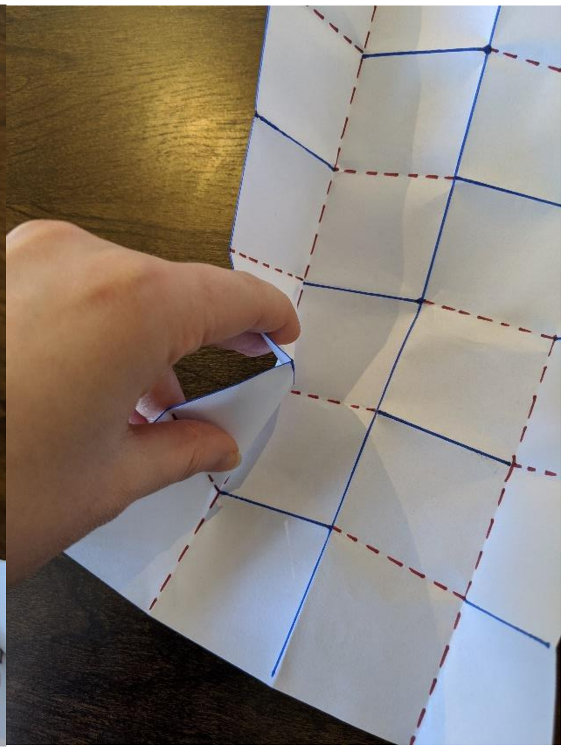
Start of the process to create your seven rows. Notice that each fold up is angled

› Unfold the paper and lay it flat. You will notice six intersecting lines running across the page (I have marked them in pen to show them clearly, you can do this too if it helps). You will again alternate mountain and valley folds using these lines as guides.



› Use your fingernail to reinforce the crease for each of the zigzags on either side of the paper, depending on whether it’s a mountain or a valley fold (see the left picture below).

› Push the zigzagging mountain folds together. As you do so, the structure should draw in on itself. This is the compressed structure of your Miura fold. **Note: This may be difficult your first time, but keep at it!**

› Grabbing the two loose corners on either side of the ‘accordion’, stretch your paper in and out to work your Miura fold. Every time that you do this, it becomes easier to return the structure to its compressed state.

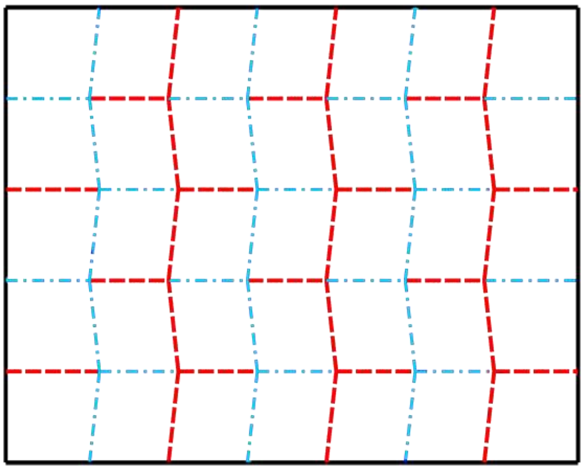
Learning Objective

› Discover how satellites are designed to fold out from a compact shape without damaging their delicate panelling.

Reflection Questions

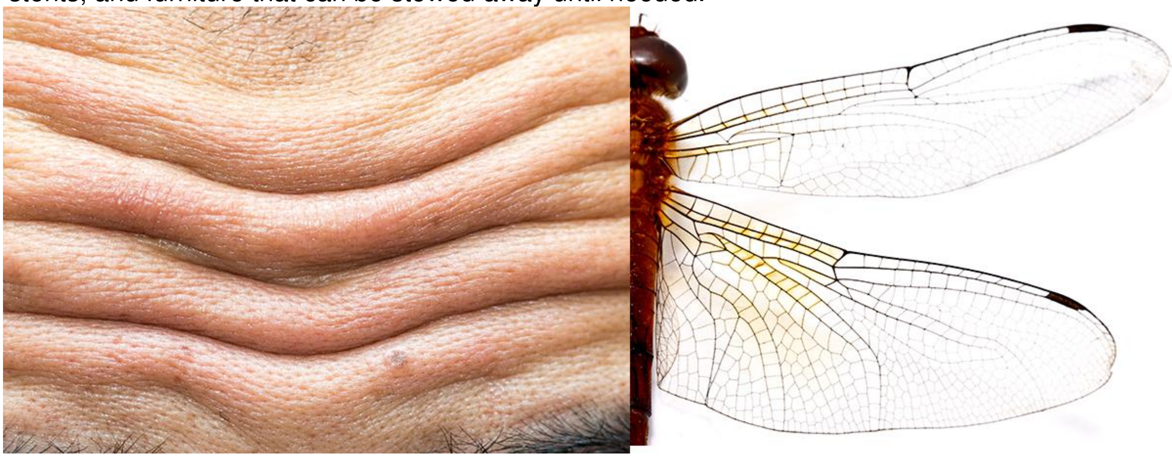
› What kind of shapes do you see caused by the folds in the flattened sheet? Where are the diagonal or straight lines?

If you noticed a pattern of interlocking polygons (any closed figure with at least three straight sides), you have identified the tessellation in the Miura fold. A tessellation is an arrangement of polygons in a repeated pattern without gaps or overlap. You may have seen art that is inspired by tessellations, like pieces created by M.C. Escher. The type of polygon that comprises the Miura fold tessellation is a parallelogram. (adaptation of Dmcq (Own work) [CC BY-SA 3.0], via Wikimedia Commons).



› Using a protractor, examine the angles of the repeated polygon. What do you notice about the angles that you have found?

Aside from the outer edges of your paper, you probably did not find any right angles. Miura did this on purpose – he proved that omitting right angles reduced the stress on the construction, thus increasing its durability. He was inspired by patterns that he had observed in nature which don't have right angles, like forehead wrinkles or the veins of a dragonfly's wing. Since then, the Miura fold has been used as inspiration for foldable solar panels, surgical stents, and furniture that can be stowed away until needed.



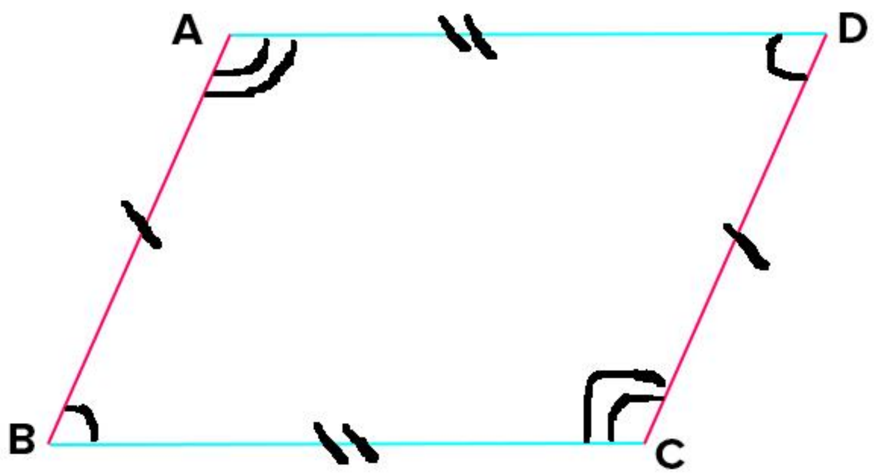
The curve of forehead wrinkles and the vein angles of dragonfly wings

You may have noted in your previous observations that opposite angles of each polygon in the tessellation are equal, which is a feature of parallelograms. The straight creases separate each parallelogram from its mirror image (reflection), while the zigzagging creases separate each parallelogram from an identical translation (copy) of itself.

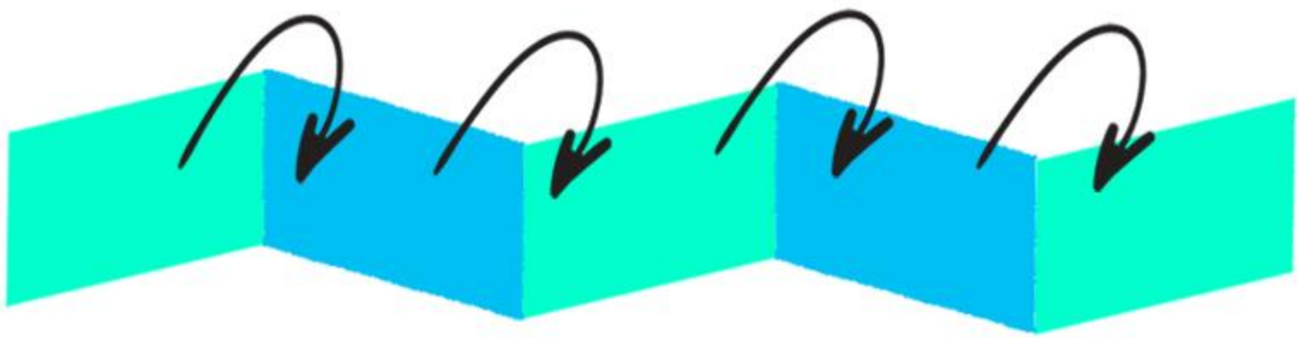
Reverse-engineer a Miura Fold

› In the previous exercise, you created a Miura fold by folding paper. In this activity, you'll make a Miura fold by first creating a tessellation of parallelograms using reflection and translation.
Create a single parallelogram to use as a template for your tessellation. Be sure to adhere to these three basic traits of parallelograms:

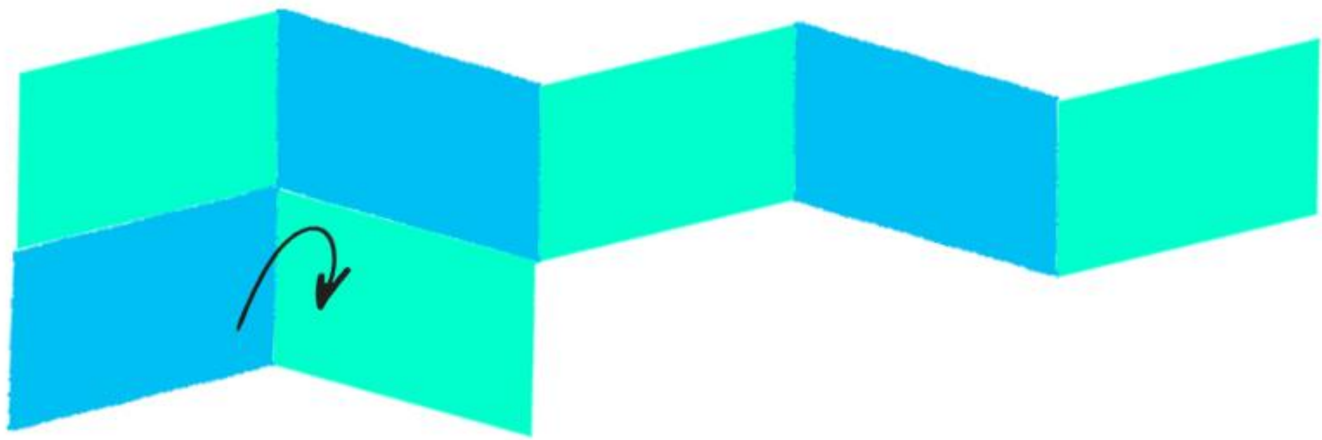
- Opposite sides are parallel and equal in length
- Opposite angles are equal
- Two consecutive angles are always supplementary (add to 180°)



› In a Miura fold, straight lines separate each parallelogram from its mirror reflection. Create a row consisting of five parallelograms across by tracing your parallelogram template that you created in step one and then reflecting it across at the straight line.



› Continue to add rows to the bottom until you have a total of seven rows.



› Cut your reverse-engineered fold by following the outer lines of your drawing.

› Create creases between each of your columns, alternating the creases on either side of the paper so you can create mountain and valley folds.

› Crease the zigzags, again alternating on either side of the paper, then push the zigzagging mountain folds together.

You can create a Miura pattern on a co-ordinate plane to practise translation and reflection. (CCSS.MATH.CONTENT.8.G.A.3, CCSS.MATH.CONTENT.HSG.CO.A.5).

Keep Experimenting with the Miura Fold

› Using a new sheet of A4 paper, change the number of rows and/or columns, and then fold the paper using the same method as the original Miura fold. Document your results. What happens to the size and angles of your parallelograms? Fold and unfold your construction a few times. Which iteration of the Miura fold has the best shape memory?

› Calculate the decimal value of the ratio of zigzagging rows to columns, and calculate the decimal value of the ratio of length to width of your paper. Compare those values. Create new Miura fold constructions that bring those values closer together and then further apart by changing the length to width ratio of your paper. For each new structure, add the ratios for rows to columns and length to width of your paper to your Miura Fold Observation Sheet. For each iteration, calculate the area of your compressed structure (when the construction is folded up) and add this to your Miura Fold Observation Sheet. What changes do you notice in the tessellation? How does the area of the compressed structure change as the ratios get closer and further apart?

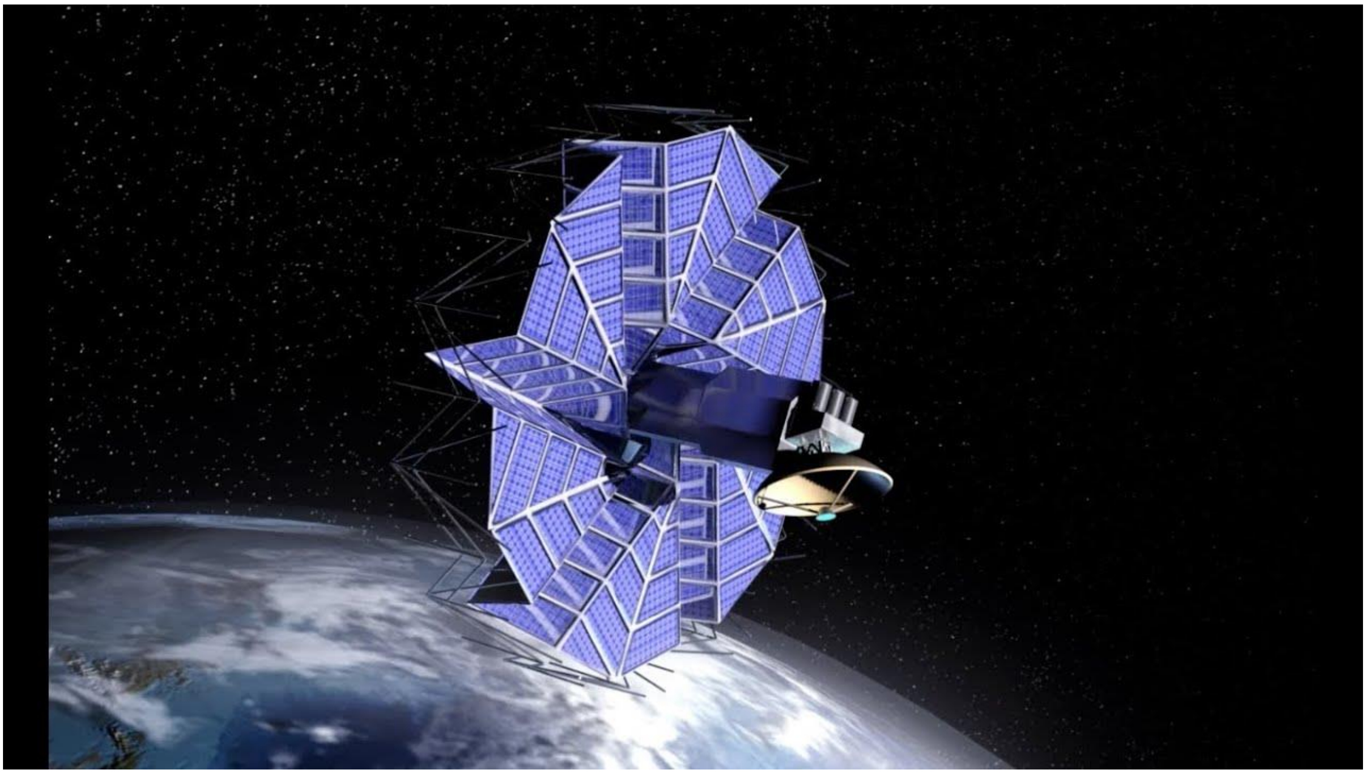
Miura Fold Observation Sheet

Row to Column Ratio	Length to Width Ratio	Area of Compressed Structure	Tessellation Changes
7/5 = **1.4**	267/210 mm = **1.27**		

› Try making Miura folds with thicker and thinner paper. What effect does the paper's thickness have on how it folds and compresses?

The Miura Fold and Tessellations in Engineering

Originally created by the Japanese astrophysicist, Koryo Miura, in 1985, this form of origami was designed to keep each piece flat. This was especially useful for structures that used rigid materials, most notably solar panels. It was for this purpose that the Miura fold was used to design the Japanese Space Flyer Unit, which launched in 1995. The satellite was able to fold into a compact shape for travel and then unfold itself once in space to allow all of the solar panels to be exposed and power the craft. This innovation influenced following space satellites from other countries to use a compact, origami inspired folding design.



NASA's folding satellite concept (https://cleantechnica.com/2015/05/12/traditional-japanese-origami-used-nasa-solar-panels/)

NASA is also using origami to design new solar panels for space. Using your new knowledge of Miura folds, give their design a go: https://www.jpl.nasa.gov/edu/learn/project/space-origami-make-your-own-starshade/

A big part of an Engineer's job is to come up with new solutions using innovative ideas. Our Engineers at Babcock are constantly trying to think outside of the box with everything that they do.